MPM 1DI Unit 8

Optimization

Day 1: Optimization of Perimeter & Area of Rectangles

TODAY:

Success Criteria:

By the end of today's lesson can you...

- determine the maximum area of a rectangle given a fixed perimeter when all four sides are enclosed.
- determine the maximum area of a rectangle given a fixed perimeter when only three sides are enclosed.
- determine the minimum perimeter of a rectangle given a fixed area.

What is "optimization"?

KEY TERM:

Optimization: the process of finding values that make a given quantity the greatest/maximum (or the least/minimum) possible amount given certain conditions.

EX. *Ian has a summer job at a fencing company. A customer has purchased 32 sections of prefabricated fencing, each 1 m in length, and wants Ian to create a rectangular pigpen with the **largest** area possible.*

This problem is an example of an optimization problem because it asks us to find the dimensions that would **maximize** the rectangular pigpen given a fixed perimeter of 32 m. This is called **optimizing** the area.

Investigation A :

How can you model the maximum area of a rectangle with a fixed perimeter?

EX. Consider the example presented on the previous page, Ian needs to find the dimensions of the rectangular pen that will **maximize** the area with a perimeter of 32 m.

Let's investigate...

1. Start by completing the table below, testing different possible dimensions. To complete the table.

   a) Determine the dimensions of 4 different rectangles that Ian could use for this fence.

   b) Calculate the area of each rectangle.

Width (m)	Length (m)	Perimeter (m)	Area (m^2)
2	14	32	$A=lw=28m^2$
4	12	32	$48m^2$
6	10	32	$60m^2$
8	8	32	$64m^2$

2. **REFLECT:** What did you find?

   a) What are the dimensions of the rectangle with the maximum, or optimal area?

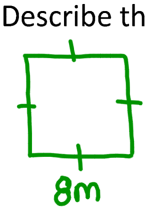
   8m × 8m

   b) What is the maximum area?

   $64m^2$

   c) What happened to the area as the length and width became closer in value? The area got bigger as the length + width got closer in value.

   d) Describe the shape of the rectangle with maximum area.



a square

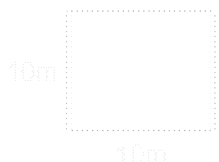
3. Suppose the customer decides to use 40 m of fencing instead of 32 m.

   a) Predict the dimensions of the rectangular pen with the maximum area.

   10m × 10 m

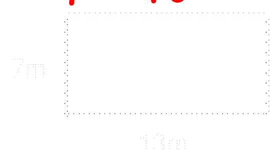
   b) Draw rectangles and find their areas to test your hypothesis.

P=40m



$A = lw$
$= 10(10)$
* $\boxed{= 100 m^2}$

P=40m



$A = lw$
$= 13(7)$
$\boxed{= 91 m^2}$

P=40m

$A = lw$
$= 4(16)$
$\boxed{= 64 m^2}$

4. **REFLECT:**

   a) What is the ideal shape for maximizing the area of a rectangle when given a fixed perimeter?

   a square (or as close to a square as possible)

   → length & width as close to equal as possible

   b) How can you predict the dimensions of a rectangle with maximum area if you know the perimeter?

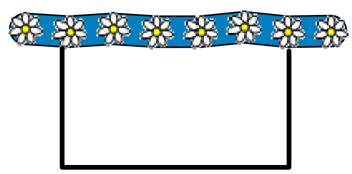
   side length = $\frac{P}{4}$

   * then all sides will be equal.

Investigation B :

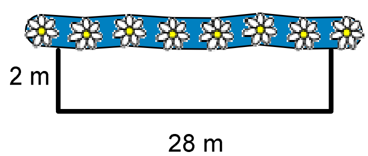
How can you model the maximum area of a rectangle with a fixed sum of the lengths of only three sides?

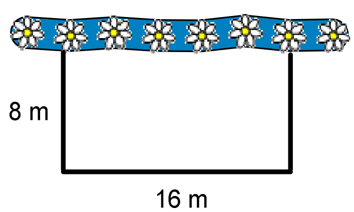
EX. Ian's customer decides to use an existing hedge as one of the boundaries for the pigpen enclosure. This means that he will only use the prefabricated fencing on three sides of the rectangular pen. The client still wants the pen to have the greatest area possible.

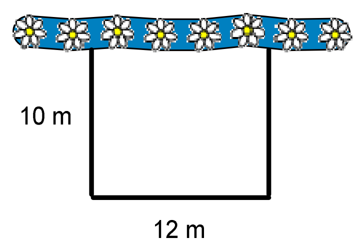


1. Ian has 32 m of prefabricated fencing.

   a) Sketch rectangles to determine the dimensions of the rectangle that has the maximum area.







   b) Record your results in the table below.

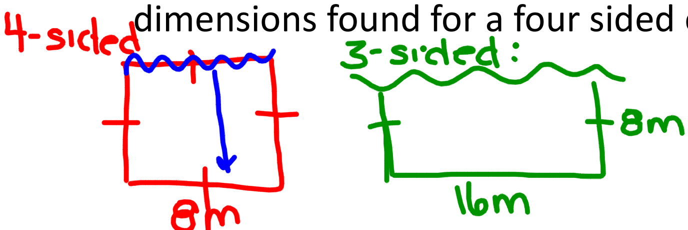
Width (m)	Length (m)	Sum of Lengths of Three Sides (m)	Area (m^2)
6	20	32	$A = lw = 120m^2$
7	18	32	$126m^2$
* 8	16	32	$128m^2$
9	14	32	$126m^2$
10	12	32	$120m^2$

2. **REFLECT:**

a) What are the dimensions of the rectangular pen with maximum area?

16m × 8m

b) Examine the length and width of the enclosure with maximum area. Do any relationships exist between these dimensions and the dimensions found for a four sided enclosure with same perimeter?



- widths are the same
- length is twice as big $(w = \frac{P}{4})$ $(l = 2 \times \frac{P}{4})$

c) Will the hedge allow Ian to enclose more, less, or the same amount of area as before?

more ($128m^2$ × $64m^2$)

→ twice as much

Summary:

When Optimizing the Area and Perimeter of a rectangle, there are two possibilities:

1. **Four-sided shape:** Make the figure in the shape of a square, with all sides equal in length.

2. **Three-sided shape:** Use an existing wall to create one side of the rectangle. For the remaining three sides, the length is twice as long as the two widths.

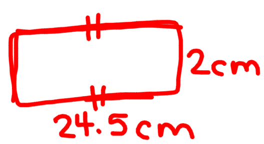
Examples:

EX. 1. a) Determine the dimensions of a rectangle with maximum area that has a perimeter of 60 m.

$$s = \frac{P}{4}$$
$$= \frac{60}{4} = 15\text{m}$$

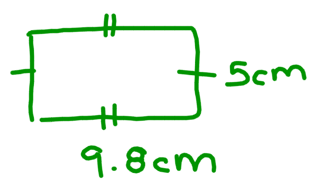
$\therefore$ the dimensions are 15m × 15m.

b) Determine the minimum perimeter of a rectangle that has an area of 49 cm².



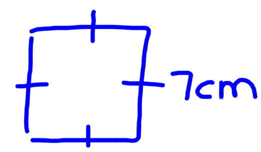
$A = 49\text{cm}^2$

$P = 53\text{cm}$



$A = 49\text{cm}^2$

$P = 29\text{cm}$



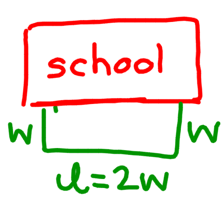
$A = 49\text{cm}^2$

* $\boxed{P = 28\text{cm}}$

*to minimize perimeter given a fixed area, the ideal shape is a square

$s = \sqrt{A}$

EX. 2. Sir Adam Beck PS is adding a rectangular kindergarten playground to the side of the school. The school will form one side of the rectangle. The area of the playground is to be 72 m². Minimizing the perimeter will minimize the cost of the fence. What dimensions use the minimum length of fence?



$A = lw$

$A = 2w \cdot w$

$A = 2w^2$

$\frac{A}{2} = w^2$

GENERAL FORMULA: how to find the side length

$w = \sqrt{\frac{A}{2}}$

$$w = \sqrt{\frac{A}{2}}$$
$$= \sqrt{\frac{72}{2}}$$
$$= \sqrt{36}$$
$$= 6$$

$l = 2w$
$= 12$

$(P = 6 + 12 + 6$
$= 24\text{m})$

$\therefore$ the dimensions are 12m × 6m

Practice: p. 487 #1, 2, 3a, 5, 6-8, 11, 12