MPM 1DI Unit 8

Optimization

Day 3: Optimization of Cylinders

RECALL:

Last class we looked at a) how to optimize (maximize) the volume of a square-based prism with a given surface area & b) how to optimize (minimize) the surface area of a square-based prism given its volume.

Success Criteria:

After last class, you should...

- be able to determine the dimensions and minimum surface area of a square-based prism given a fixed volume.
- be able to determine the dimensions and maximum volume of a square-based prism given a fixed surface area.

TODAY:

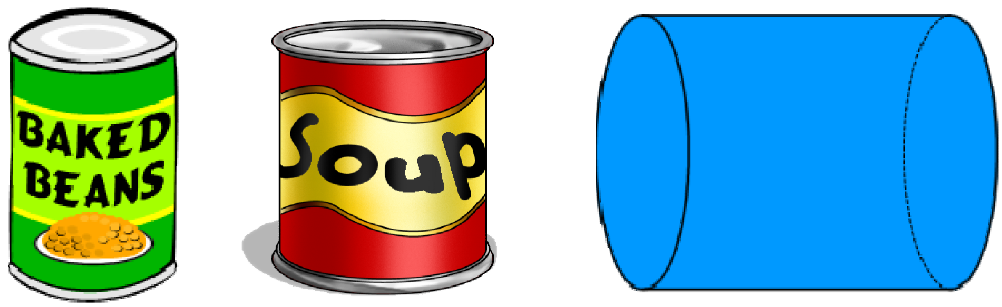
Success Criteria:

By the end of today's lesson can you...

- determine the minimum surface area of a cylinder given a fixed volume.
- determine the maximum volume of a cylinder given a fixed surface area.

Investigation A :

How can you compare the volumes of cylinders with the same surface area?



Many products come in cylinders. Your task is to design a cylindrical juice can that uses no more than 375 cm^2 of aluminum. The can should have the greatest capacity possible.

Let's investigate...

1. Complete the table below.

NOTE: To investigate the volume of the cylinder as its radius changes, it will help to have an expression for the height in terms of the radius given that the surface area is 375 cm².

$SA = 2\pi r^2 + 2\pi r h$

$375 = 2\pi r^2 + 2\pi r h$

$\frac{375 - 2\pi r^2}{2\pi r} = \frac{2\pi r h}{2\pi r}$

$h = \frac{375 - 2\pi r^2}{2\pi r}$

Radius (cm)	Height (cm)	$V=\pi r^2 h$ Volume (cm³)	Surface Area (cm^2)
1	58.7	184.4	375
2	27.8	349.3	375
3	16.9	477.8	375
* 4	10.9	547.9	375
5	6.9	541.9	375
6	3.9	441.1	375
7	1.5	230.9	375

2. a) What is the maximum volume for the cans in your table?

$547.9\ cm^3$

b) What are the radius and height of the can with this volume?

radius = 4cm
height = 10.9cm

REFLECT: Summarize your findings.

a) Do any relationships exist between the radius and height of a cylinder with maximum volume for a given surface area?

ideal dimensions: $h = d$, $\therefore h = 2r$

b) Do the dimensions found in the investigation give the optimal volume for the surface area of 375 cm²?

No, not quite because $10.9 \neq 2(4)$

c) Can we determine the dimensions of a can with a volume greater than the value in the table?

$SA = 2\pi r^2 + 2\pi r h$

$SA = 2\pi r^2 + 2\pi r(2r)$

$SA = 2\pi r^2 + 4\pi r^2$

$\frac{SA}{6\pi} = \frac{6\pi r^2}{6\pi}$

$\sqrt{r^2} = \sqrt{\frac{SA}{6\pi}} \rightarrow r = \sqrt{\frac{SA}{6\pi}}$

$r = \sqrt{\frac{375}{6\pi}}$

$= 4.46\ cm$

$h = 2r$

$= 8.92\ cm$

$\therefore$ the optimal dimensions are $r = 4.46$cm + $h = 8.92$cm

d) How can you predict the dimensions of a cylinder with maximum volume if you know the surface area? Can we come up with a general formula?

$r = \sqrt{\frac{SA}{6\pi}}$, $h = 2r$

EX. 1. Maximize the Volume of a Cylinder

a) Determine the dimensions of the cylinder with maximum volume that can be made with 600 cm^2 of aluminum. Round the dimensions to the nearest hundredth of a centimetre.

$$r = \sqrt{\frac{SA}{6\pi}}$$
$$= \sqrt{\frac{600}{6\pi}}$$
$$\doteq 5.64\text{cm}$$

$$h = 2r$$
$$= 2(5.64)$$
$$= 11.28\text{ cm}$$

$\therefore$ radius = 5.64cm and height is 11.28cm

b) What is the volume of this cylinder to the nearest cubic centimetre?

$$V = \pi r^2 h$$
$$= \pi(5.64)^2(11.28)$$
$$\therefore V = 1127\text{ cm}^3$$

Investigation B :

How can you compare the surface areas of cylinders with the same volume?

Your task is to design a cylindrical juice can that must hold 1000 mL of juice.

1. Complete the table below.

NOTE: To investigate the surface area of the cylinder as its radius changes, it will help to have an expression for the height in terms of the radius given that the volume is 1000 mL.

$$\frac{V}{\pi r^2} = \frac{\pi r^2 h}{\pi r^2}$$

$$h = \frac{1000}{\pi r^2}$$

$2\pi r^2 + 2\pi r h$

Radius (cm)	Height (cm)	*Volume* ($1cm^3 = 1mL$)	*Surface Area* (cm^2)
1	318.3	1000	2006.2
2	79.6	1000	1025.4
3	35.4	1000	723.8
4	19.9	1000	600.7
* 5	12.7	1000	556.1
6	8.8	1000	557.9
7	6.5	1000	593.8

2. a) What is the minimum surface area for the cans in your table?

$556.1\ cm^2$

b) What are the radius and height of the can with this surface area?

radius = 5cm
height = 12.7cm

REFLECT: Summarize your findings.

a) Do any relationships exist between the radius and height of a cylinder with minimum surface area for a given volume?

$h = 2r$ $(=d)$ height equals diameter

b) Do the dimensions found in the investigation give the optimal surface area for the volume of 1000 mL?

$12.7 \neq 2(5)$

No, it is not optimal because $h \neq 2r$

c) Can we determine the dimensions of a can with a surface area smaller than the value in the table?

$V = \pi r^2 h$

$V = \pi r^2 (2r)$

$\frac{V}{2\pi} = \frac{2\pi r^3}{2\pi}$

$r^3 = \frac{V}{2\pi}$

$r = \sqrt[3]{\frac{V}{2\pi}}$

$r = \sqrt[3]{\frac{1000}{(2\pi)}}$

$= \sqrt[3]{159.2}$

$\boxed{r \doteq 5.4\ cm \quad h = 10.8\ cm}$

d) How can you predict the dimensions of a cylinder with minimum surface area if you know the volume? Can we come up with a general formula?

$r = \sqrt[3]{\frac{V}{2\pi}} \rightarrow h = 2r$

EX. 2. Minimize the Surface Area of a Cylinder

a) Determine the least amount of aluminum required to construct a cylindrical container with a 4L capacity, to the nearest tenth of a square centimetre.

RECALL: 1 mL = 1 cm^3 and because 1L = 1000 mL, therefore 1L = 1000cm^3

Step 1: Figure out the dimensions → need r + h

$4L = 4000 mL$
$= 4000 cm^3$

$r = \sqrt[3]{\frac{V}{2\pi}}$
$= \sqrt[3]{\frac{4000}{(2\pi)}}$
$\doteq 8.6 cm$

$h = 2r$
$= 2(8.6)$
$= 17.2 cm$

Step 2: Calculate surface area.

$SA = 2\pi r^2 + 2\pi r h$
$= 2\pi(8.6)^2 + 2\pi(8.6)(17.2)$
$= 1394.1 cm^2$

$\therefore$ need $1394.1 cm^2$ of aluminum.

b) Describe any assumptions you made.

No overlap

Practice: p. 508 #1-4 & p. 513 #1, 2, 5, 6